Chapter 3: Planning and Scheduling

Resolving Conflict via Coloring

Special Topics
Goals of Three Types of Scheduling Problems

- **Optimization issues** – Try to maximize profit, minimize cost.
  - Example: Scheduling machine time for earliest completion time

- **Equity** – Try to make things fair for all participants.
  - Example: Schedule baseball games (same number home and away games)

- **Conflict Resolution** – Try to prevent conflicts from happening.
  - Example: Scheduling college final examinations for end of term
Resolving Conflict via Coloring

- **Vertex Coloring** – The vertex coloring problem for a graph requires assigning each vertex of the graph a color (label), such that two vertices joined by an edge are assigned different colors.

- **Chromatic Number** – The chromatic number is the minimum number of colors needed to label the vertices of a graph so that no two vertices of the graph joined by an edge get the same color.
Scheduling Exams Using Vertex Coloring

- There are 8 finals to schedule with only 2 air-conditioned rooms.

- In the graph, courses are represented by vertices and two course are joined by an edge for every student enrolled in both courses.
This is a chart of the final exams. X’s indicate courses that have shared students who might end up having finals scheduled at the same time (conflicts).

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Example

- We want to accomplish two things:
- Minimize the time slots for final exams.
- Fit the final exams into the available rooms.
This graph represents conflict info about the courses. Courses are vertices while edges indicate shared students.
The vertices are colored with 8 colors. No two edges have the same color, so there are no conflicts. Each color represents a time slot. Since there are 8 time slots, this is not optimal. You only need one room, however.
This graph shows a reduction from 8 colors to 4 colors, which means there are now four time slots needed. This was accomplished through trial and error. Most tests now need two rooms or fewer (red, purple, and violet) but now three rooms are needed for the green time slot 2). There might only be two rooms available.
Now there are 4 colors with four time slots. Each color appears twice so only need two rooms are needed.

The chromatic number is 4 as in d (we scheduled 8 exams in 4 time slots without a conflict).
Realistic problems in scheduling government committees, high school and university final examinations, job interviews are usually so large that graph coloring algorithms have to be incorporated into elaborate software packages to solve them. As with all of the other Management Science problems we learned in Chapter 3, we only touch on the surface of usually complex problems.
Resolving Conflict – Assignment

- HW: pages 97–100 #67–83 odds