# 1st Six Weeks ToM 2012-2013

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Worksheet – Scheduling
Special Topics

Use the following digraph to complete schedules for the following priority lists. Use a ruler and be neat.

1. Using the priority list $T_4, T_3, T_9, T_{10}, T_8, T_6, T_1, T_7, T_2$ schedule the project with two processors.

2. Using the priority list $T_2, T_4, T_6, T_8, T_{10}, T_1, T_3, T_5, T_7, T_9$ schedule the project with two processors.

3. Using the priority list $T_4, T_3, T_9, T_{10}, T_8, T_5, T_6, T_1, T_7, T_2$ schedule the project with three processors.

4. Using the priority list $T_2, T_4, T_6, T_8, T_{10}, T_1, T_3, T_5, T_7, T_9$ schedule the project with three processors.
Worksheet - Scheduling
Special Topics

Use the following digraph to complete schedules for the following priority lists. Use a ruler and be neat.

1. Using the priority list \( T_4, T_3, T_9, T_{10}, T_8, T_5, T_6, T_1, T_7, T_2 \) schedule the project with two processors.

2. Using the priority list \( T_2, T_4, T_6, T_8, T_{10}, T_1, T_3, T_5, T_7, T_9 \) schedule the project with two processors.

3. Using the priority list \( T_4, T_3, T_9, T_{10}, T_8, T_5, T_6, T_1, T_7, T_2 \) schedule the project with three processors.

4. Using the priority list \( T_2, T_4, T_6, T_8, T_{10}, T_1, T_3, T_5, T_7, T_9 \) schedule the project with three processors.
Chromatic number The chromatic number of a graph G is the minimum number of colors (labels) needed in any vertex coloring of G. (p. 85)

Critical-path scheduling A heuristic algorithm for solving scheduling problems where the list-processing algorithm is applied to the priority list obtained by listing next in the priority list a task that heads a longest path in the order-requirement digraph. This task is then deleted from the order-requirement digraph, and the next task placed in the priority list is obtained by repeating the process. (p. 75)

Decreasing-time-list algorithm The heuristic algorithm that applies the list-processing algorithm to the priority list obtained by listing the tasks in decreasing order of their time length. (p. 79)

First-fit (FF) A heuristic algorithm for bin packing in which the next weight to be packed is placed in the lowest-numbered bin already opened into which it will fit. If it fits in no open bin, a new bin is opened. (p. 81)

First-fit decreasing (FFD) A heuristic algorithm for bin packing where the first-fit algorithm is applied to the list of weights sorted so that they appear in decreasing order. (p. 82)

Heuristic algorithm An algorithm that is fast to carry out but that doesn't necessarily give an optimal solution to an optimization problem. (p. 81)

Independent tasks Tasks are independent when there are no edges in the order-requirement digraph. These are tasks that can be performed in any order. (p. 78)

List-processing algorithm A heuristic algorithm for assigning tasks to processors: Assign the first ready task on the priority list that has not already been assigned to the lowest-numbered processor that is not working on a task. (p. 69)

Machine-scheduling problem The problem of assigning tasks to processors so as to complete the tasks by the earliest time possible. (p. 67)

Next-fit (NF) A heuristic algorithm for bin packing in which a new bin is opened if the weight to be packed next will not fit in the bin that is currently being filled; the current bin is then closed. (p. 81)

Next-fit decreasing (NFD) A heuristic algorithm for bin packing where the next-fit algorithm is applied to the list of weights sorted so that they appear in decreasing order. (p. 82)

Priority list An ordering of the collection of tasks to be scheduled for the purpose of attaining a particular scheduling goal. One such goal is minimizing completion time when the list-processing algorithm is applied. (p. 68)

Processor A person, machine, robot, operating room, or runway with time that must be scheduled. (p. 67)

Ready task A task is called ready at a particular time if its predecessors, as given by the order-requirement digraph, have been completed by that time. (p. 69)

Vertex coloring A vertex coloring of a graph G is an assignment of labels, which can be thought of as "colors," to the vertices of G so that vertices joined by an edge get different labels (colors). (p. 85)

Worst-case analysis The study of the list-processing algorithm (more generally, any algorithm) from the point of view of how well it performs on the hardest problems it may be used on. See also average-case analysis. (p. 78)

Worst-fit (WF) A heuristic algorithm for bin packing in which the next weight to be packed is placed into the open bin with the largest amount of room remaining. If the weight fits in no open bin, a new bin is opened. (p. 81)

Worst-fit decreasing (WFD) A heuristic algorithm for bin packing where the worst-fit algorithm is applied to the list of weights sorted so that they appear in decreasing order. (p. 82)

**SKILLS CHECK**

1. What is the minimum time required to complete 8 independent tasks with a total task time of 64 minutes on 4 machines?
   (a) Less than 8 minutes
   (b) Between 8 and 10 minutes
   (c) More than 12 minutes

2. Given the order-requirement digraph below (time in minutes) and the priority list T1, T2, T3, T5, T6, apply the list-processing algorithm to construct a schedule using two processors. The completion time of the resulting schedule is

   ![Diagram](image)

3. The following digraph cannot be an order-requirement digraph because
   (a) no vertex has three edges that enter that particular vertex.
   (b) it has a directed circuit.
   (c) all the tasks require the same time to complete.
4. Suppose that a crew can complete in a minimum amount of time the job whose order-requirement digraph is shown below. If task $T_2$ is shortened from 5 minutes to 2 minutes, then the maximum amount by which the completion time for the entire job can be shortened is 

![Diagram](image)

5. Suppose that independent tasks require a total of 30 minutes, while only one task takes as long as 10 minutes. If these tasks are scheduled on two machines, 
(a) the tasks might take longer than 16 minutes to complete.
(b) the tasks can never take longer than 15 minutes to complete.
(c) the tasks can always be completed within 16 minutes.

6. The subscripts for the tasks that make up a critical path for the order-requirement digraph below are: ____, _____, _____, _____, _____.

![Diagram](image)

7. Which statement is true for the following digraph?
(a) This digraph cannot be the order-requirement digraph for a scheduling problem because the digraph has no (directed) edges.
(b) This digraph cannot be the order-requirement digraph for a scheduling problem because it is not allowed for all the tasks to have the same time length.
(c) This digraph can be the order-requirement digraph for a scheduling problem.

8. The subscripts for the tasks in a critical path list associated with the following order-requirement digraph are: ____, _____, _____, _____, _____.

![Diagram](image)

9. Assume an order-requirement digraph has a critical path with length 20 minutes. Based on this information, when the tasks are scheduled on two machines, how much time will be required?
(a) Exactly 10 minutes
(b) Exactly 20 minutes
(c) At least 20 minutes

10. The list-processing algorithm is used to schedule independent tasks lasting 6, 7, 4, 3, and 6 minutes on three machines, using these times as given for a list. The completion time for all the tasks will be ________.

11. Assume a job consists of six independent tasks ranging in time from 2 to 10 minutes and totaling 27 minutes. Efficiently scheduled on three machines, how much time will the job require?
(a) Exactly 9 minutes
(b) Exactly 10 minutes
(c) More than 10 minutes

12. When the decreasing-time-list algorithm is used to schedule independent tasks lasting 6 minutes, 7 minutes, 4 minutes, 3 minutes, and 6 minutes, on two machines, a schedule results where the tasks are completed after ______ minutes.

13. A radio announcer has 10 songs of various lengths to schedule into several segments. The announcer must identify the station at least once every 15 minutes, so the segments cannot be longer than 15 minutes. This job can be solved using the
(a) list-processing algorithm for independent tasks.
(b) critical-path scheduling algorithm.
(c) first-fit algorithm for bin packing.

14. Use the first-fit (FF) bin-packing algorithm to pack the following weights into bins that can hold no more than 10 lb: 6 lb, 7 lb, 4 lb, 3 lb, 6 lb. The number of bins required is ________.

15. Use the worst-fit-decreasing (WFD) bin-packing algorithm to pack the following weights into bins that can hold no more than 10 lb: 6 lb, 7 lb, 4 lb, 3 lb, 6 lb. How many bins are holding a full 10 lb?
(a) 0 bins
(b) 1 bin
(c) 2 bins
16. The first-fit decreasing (FFD) bin-packing algorithm is applied to the weight list 1, 2, 3, 4, 5, 5, 6, 8 for packing into bins of capacity 10. The item of weight 2 is packed into the bin numbered __ when the packed bins are numbered from left to right.

17. A vertex coloring seeks to color the vertices of a graph to ensure which of the following traits?
   (a) Every color is used.
   (b) Every edge connects vertices of the same color.
   (c) Vertices of the same color are never connected by an edge.

18. Assume the 8 corners of a cube represent vertices of a graph and the 12 edges of a cube represent the cube's edges. The chromatic number of this graph is __.

19. The minimum number of colors needed to color the vertices of the accompanying graph is __.

(a) 4.
(b) 2.
(c) 3.

20. A graph that has a circuit of length 3 can always be vertex colored with no fewer than __ colors.

CHAPTER 3 EXERCISES

3.1 Scheduling Tasks

3.2 Critical-Path Schedules

1. List as many scheduling situations as you can for these environments:
   (a) Your school
   (b) Hospital
   (c) Train station
   (d) Police station
   (e) Bookstore
   (f) Internet café
   (g) Firehouse
   (h) Television studio

2. Compare and contrast the scheduling problems which arise at a
   (a) Fast food restaurant.
   (b) Standard sit-down restaurant.

3. You and your two housemates are planning to have a party this Friday night at your apartment. Eight guests are expected and there are plans to serve a small homemade dinner. List the tasks involved in carrying out such a party, and the types of processors to be used to carry out the tasks. Can any of the tasks be done simultaneously?

4. Jane is planning a getaway weekend at a ski resort. She plans to leave work in Manhattan at 1 P.M. and must make her way to a local airport for a 5 P.M. shuttle plane to Boston. She then hopes to get a bus to the nearby resort. Discuss the tasks that Jane must complete to be at the resort by 10 P.M. What are the different types of processors involved in getting these tasks done? Can any of these tasks be done simultaneously?

5. Use the list-processing algorithm to schedule the tasks in the following order-requirement digraph on

6. Consider the following order-requirement digraph:

(a) Find the length of the critical path.
(b) Schedule these seven tasks on two processors using the list algorithm and the lists:
   (i) \( T_1, T_2, T_3, T_4, T_5, T_6, T_7 \)
   (ii) \( T_2, T_1, T_3, T_6, T_5, T_4, T_7 \)
(c) Does either list lead to a completion time that equals the length of the critical path?
(d) Show that no list can ever lead to a completion time equal to the length of the critical path (providing the schedule uses two processors).

7. (a) Use the following order-requirement digraph to schedule the 6 tasks $T_1, T_2, T_3, T_4, T_5, T_6$ on two processors with the priority lists:
   (i) $T_1, T_2, T_3, T_4, T_5, T_6$
   (ii) $T_1, T_6, T_3, T_5, T_4, T_2$

(b) Are either of the schedules produced from these lists optimal? If not, can you find a priority list that will result in an optimal schedule?
(c) Find the critical path and its length. Explain why no schedule has earliest completion time equal to the length of the critical path.

8. (a) Repeat Exercise 7, but interchange the task times of tasks $T_2$ and $T_6$.
(b) How does the completion time for an optimum schedule for this situation compare with the optimum schedule for Exercise 7?

9. (a) If one adds a new directed edge to an order-requirement digraph $D$, can the critical path in the the new order-requirement digraph $D'$ have longer length?
(b) If one adds a new directed edge to an order-requirement digraph $D$, can the critical path in the the new order-requirement digraph $D'$ have shorter length?

10. Discuss scheduling problems for which it is not reasonable to assume that once a processor starts a task, it will always complete that task, before it works on any other task. Give examples for which this approach would be reasonable.

11. For the accompanying order-requirement digraph, apply the list-processing algorithm, using three processors for lists (a) through (c). How do the completion times obtained compare with the length of the critical path?

12. (a) Can you find an order-requirement digraph with four tasks for which every priority list used to schedule the tasks on two machines assigns task $T_4$ to machine 1 at time 0?
(b) Can you choose the order-requirement digraph in part (a) so that machine 2 stays idle for all lists from time 0 to time 3?

13. Can you give examples of scheduling problems for which it seems reasonable to assume that all the task times are the same?

14. Use the list-processing algorithm to schedule the tasks in the following order-requirement digraph on

(a) two processors using the list $T_1, T_2, T_3, T_4, T_5, T_6, T_7$
(b) two processors using the list $T_1, T_3, T_5, T_7, T_2, T_4, T_6, T_8$
(c) three processors for lists (a) through (c). How do the completion times obtained compare with the length of the critical path?

15. Can you find a list that gives rise to the optimal schedule shown in Figure 3.14 for the order-requirement digraph in Figure 3.12?

16. Consider the following order-requirement digraph:

(a) Find the critical path(s).
(b) Schedule these tasks on one processor using the critical-path scheduling method.

(c) Schedule these tasks on one processor using the priority list obtained by listing the tasks in order of decreasing time.

(d) Does either of these schedules have idle time? How do their completion times compare?

(e) If two different schedules have the same completion time, what criteria can be used to say one schedule is superior to the other?

(f) Schedule these tasks on two processors using the order-requirement digraph shown and the priority list from part (b).

(g) Does the schedule produced in part (f) finish in half the time that the schedule in part (b) did, which might be expected, since the number of processors has doubled?

(h) Schedule the tasks on (i) one processor and (ii) two processors (using the decreasing-time list), assuming that each task time has been reduced by one. Do the changes in completion time agree with your expectations?

17. (a) Can all the processors being used to schedule tasks be simultaneously idle at a time before the completion time of a collection of tasks scheduled using the list-processing algorithm?

(b) Explain why the list-processing algorithm cannot give rise to the schedule below, regardless of what priority list was used to schedule the tasks on the two processors.

18. To prepare a meal quickly involves carrying out the tasks shown (time lengths in minutes) in the following order-requirement digraph:

(a) If Mike prepares the meal alone, how long will it take?

(b) If Mike can talk Mary into helping him prepare the meal, how long will it take if the tasks are scheduled using the list $T_5, T_9, T_1, T_7, T_2, T_6, T_8, T_4, T_7$ and the list-processing algorithm?

(c) If Mike can talk Mary and Jack into helping him prepare the meal, how long will it take if the tasks are scheduled using the same list as in part (b)?

(d) What would be a reasonable set of criteria for choosing a priority list in this situation?

19. (a) Making use of the order-requirement digraph below, determine at time 0 which tasks are ready.

(b) What is special about tasks $T_1$ and $T_6$?

(c) What is the critical path, and what is its length?

(d) Schedule the tasks on three processors with the priority list $T_1, \ldots, T_6$.

(e) Is the schedule found in part (d) optimal?

(f) Schedule the tasks on three processors using the priority list $T_6, \ldots, T_1$.

(g) Is the schedule found in part (f) optimal?

(h) Can you find a priority list that yields an optimal schedule?

20. (a) In Exercise 19, what priority list would be used if you applied the critical-path scheduling method?

(b) Use this priority list to schedule the tasks on three processors. Is this schedule optimal?

(c) How does this schedule compare with the schedules that you found using the lists in Exercise 19?

21. Consider the order-requirement digraph below. Suppose one plans to schedule these tasks on two identical processors.

(a) How many different priority lists can be used to schedule the tasks?

(b) Can all these priority lists lead to different schedules? If not, why not?

(c) Can an optimal schedule have no idle time? Can you give two different reasons why an optimal schedule must have some idle time?
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(d) Is there any list that produces a schedule where the second processor has no idle time?

22. (a) In Exercise 21, how many different lists are there that do not list T1 first?
(b) Would it make any sense not to list T1 first in a list?
(c) Construct a list and schedule the tasks on two processors.
(d) Can you find another list that leads to a different completion time than the schedule you found for part (c)?
(e) Find a list that leads to an optimal schedule.

23. Can you find an order-requirement digraph with five tasks for which every possible list yields exactly the same schedule?

24. Can you find an order-requirement digraph involving three tasks such that the schedule corresponding to every list is different?

25. At a large toy store, scooters arrive unassembled in boxes. To assemble a scooter, the following tasks must be performed:
Task 1. Remove parts from the box.
Task 2. Attach wheels to the footboard.
Task 3. Attach vertical housing.
Task 4. Attach handlebars to vertical housing.
Task 5. Put on reflector tape.
Task 6. Attach bell to handlebars.
Task 7. Attach decals.
Task 8. Attach kickstand.
Task 9. Attach safety instructions to handlebars.
(a) Give reasonable time estimates for these tasks and construct a reasonable order-requirement digraph. What is the earliest time by which these tasks can be completed?
(b) Schedule this job on two processors (humans) using the decreasing-time-list algorithm.

26. If two schedules for the same number of processors have the same completion time, can one schedule have more idle time than the other?

27. Could the schedule below be obtained by applying the list-scheduling algorithm to some order-requirement digraph?

3.3 Independent Tasks

28. Could the following schedule be obtained by applying the list-scheduling algorithm to some order-requirement digraph?

29. For the following schedules, can you produce a list so that the list-processing algorithm produces the schedule shown when the tasks are independent? What are the task times for each task?

30. Once an optimal schedule has been found for independent tasks (see diagrams in Exercise 29), usually the scheduling of the tasks can be rearranged and the same optimal time achieved.

31. The task times of eight independent tasks T1 to T8 are 1, 2, 3, 4, 5, 6, 7, 8.
(a) Schedule the tasks on two processors using the lists (i) T1, T2, ..., T8 and (ii) T8, T7, ..., T1.
(b) Is either of the schedules you get in part (a) optimal? If not, find a list that gives an optimal schedule.

32. Repeat Exercise 31, but schedule the tasks (with the same lists) on three processors. If the schedules you get are not optimal, find a list that gives an optimal schedule.

33. Discuss different criteria that might be used to construct a priority list for a scheduling problem.

34. Some scheduling projects have due dates for tasks (times by which a given task should be completed) and release dates (times before which a task cannot have work begun on it). Give examples of circumstances where these situations might arise.

35. Using the lists you found in Exercise 29 and the task times you computed for those independent tasks, schedule the tasks for (a) on four processors and the
tasks for (b) on five processors. Can you see why for any schedule you may produce for (a) on four processors and (b) on five processors there must be some idle time for one or more processors?

36. Given the following order-requirement digraph:

(a) Use the list-processing algorithm to schedule these seven tasks on two processors using these lists:
   (i) \( T_1, T_3, T_7, T_5, T_4, T_6, T_2 \)
   (ii) \( T_1, T_3, T_7, T_5, T_6, T_4, T_2 \)
   (iii) The list obtained by listing the tasks in order of decreasing time

(b) Try to determine if any of the resulting schedules are optimal.

(c) Schedule the tasks using the critical-path scheduling method. Try to determine if this schedule is optimal.

37. Repeat the questions in Exercise 36 using the order-requirement digraph obtained by erasing all the (directed) edges shown there. How do the schedules you get compare with the ones you originally got?

38. (a) Find the completion time for independent tasks of length 8, 11, 17, 14, 19, 5, 16, 1, 18, 5, 3, 7, 6, 2, 1 on two processors, using the list-processing algorithm.

(b) Find the completion time for the tasks in part (a) on two processors, using the decreasing-time-list algorithm.

(c) Does either algorithm give rise to an optimal schedule?

(d) Repeat for tasks of lengths 19, 19, 20, 20, 1, 1, 2, 2, 3, 3, 5, 5, 11, 11, 17, 18, 18, 17, 2, 1, 6, 16, 2.

39. Repeat parts (a)-(c) of Exercise 38 for independent tasks of lengths 19, 19, 20, 20, 1, 1, 2, 2, 3, 3, 5, 5, 11, 11, 17, 18, 18, 17, 2, 16, 16, 2.

40. Suppose that independent tasks require a total of 36 minutes, while only one of the tasks takes as long as 12 minutes. If these tasks are scheduled on two machines, show by an example that the earliest completion time may be as long as 22 minutes.

41. A photocopy shop must schedule independent batches of documents to be copied. The times for the different sets of documents are (in minutes): 12, 23, 32, 13, 24, 45, 23, 23, 14, 21, 34, 53, 18, 63, 47, 25, 74, 23, 43, 43, 16, 16, 76.

(a) Construct a schedule using the list-processing algorithm on three machines.

(b) Construct a schedule using the list-processing algorithm on four machines.

(c) Repeat parts (a) and (b), but use the decreasing-time-list algorithm.

(d) Suppose union regulations require that an 8-minute rest period be allowed for any photocopy task over 45 minutes. Use the decreasing-time-list algorithm, with the preceding times modified to take into account the union requirement, to schedule the tasks on three human-operated machines.

42. Find a list that produces the following optimal schedule when the list-processing algorithm is applied to this list. (Assume the tasks are independent.)

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<th>Machine</th>
<th>1</th>
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<td>0</td>
<td>( T_2 )</td>
<td>( T_5 )</td>
<td>( T_6 )</td>
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<td>10</td>
<td>( T_5 )</td>
<td>( T_6 )</td>
<td>( T_4 )</td>
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<td>20</td>
<td>( T_1 )</td>
<td>( T_2 )</td>
<td>( T_7 )</td>
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What completion time and schedule are obtained when the decreasing-time-list algorithm is applied to this list?

43. Can you think of situations other than those mentioned in the text where scheduling independent tasks on processors occurs?

44. Can you think of real-world scheduling situations in which all the tasks have the same time and are independent? Find an algorithm for solving this problem optimally. (If there are \( n \) independent tasks of time length \( k \), when will all the tasks be finished?)

45. Show that when tasks to be scheduled are independent, the critical-path method and the decreasing-time-list method are identical.

3.4 Bin Packing

46. Two wooden wall systems are to be made of pieces of wood with lengths shown in the accompanying diagram. If wood is sold in 10-foot planks and can be cut with no waste, what number of boards would be purchased if one uses the first-fit-decreasing, next-fit-decreasing, and worst-fit-decreasing heuristics, respectively?

In solving this problem, does it make a difference if the 10-foot horizontal shelves and 6-foot vertical boards employ single-length pieces as compared with using pieces of boards that add up to 10- and 6-foot lengths?
47. It takes 4 seconds to photocopy one page. Manu- scripts of 10, 8, 15, 24, 22, 24, 20, 14, 19, 12, 16, 30, 15, and 16 pages are to be photocopied. How many photocopy machines would be required, using the first-fit-decreasing algorithm, to guarantee that all manuscripts are photocopied in 2 minutes or less? Would the solution differ if worst-fit decreasing were used?

48. A radio station’s policy allows advertising breaks of no longer than 2 minutes, 15 seconds. Using first-fit and first-fit-decreasing algorithms, determine the number of breaks into which the following ads will fit (lengths given in seconds): 80, 50, 50, 40, 60, 90, 90, 50, 40, 20, 30, 15, and 16 pages are to be photocopied. How many can you find the optimal solution? Do the same for these ad lengths: 50, 40, 40, 40, 50, 50, 30, 20, 30, 30.

49. Fiberglass insulation comes in 36-inch precut sections. A plumber must install insulation in a basement on piping that is interrupted often by joints. The distances between the joints on the stretches of pipe that must be insulated are 12, 15, 16, 12, 9, 11, 15, 17, 12, 14, 17, 18, 19, 21, 31, 7, 21, 24, 15, 16, 12, 9, 8, 27, 22, 18 inches. How many precut sections would he have to use to provide the insulation if he bases his decision on
(a) next-fit?
(b) next-fit decreasing?
(c) worst-fit?
(d) worst-fit decreasing?

50. The files that a company has for its employees dealing with utilities occupy 100, 120, 60, 90, 110, 45, 30, 70, 60, 50, 40, 25, 65, 25, 55, 35, 45, 60, 75, 30, 120, 100, 60, 90, 85 sectors. If, after operating systems are installed, a disk can store up to 480 sectors, determine the number of disks needed to store the utilities if each of these heuristics is used to pack the disk with files:
(a) next-fit?
(b) next-fit decreasing
(c) first-fit
(d) first-fit decreasing

51. Advertisements for the TV show Q are permitted to last up to a total of 8 minutes, and each group of ads can last up to 2 minutes. If the ads slated for Q last 63, 32, 11, 19, 24, 87, 64, 36, 27, 42, 63 seconds, determine if FF and FFD yield acceptable configurations for the ads.

52. Consider the heuristic for packing bins known as best-fit described as follows: Keep track of how much room remains in each unfilled bin and put the next item to be packed into that bin that would leave the least room left over after the item is put into the bin. (For example, suppose that bin 4 had 6 units left, bin 7 had 5 units left, and bin 9 had 8 units left. If the next item in the list had size 5, then first-fit would place this item in bin 4, worst-fit would place the item in bin 9, while best-fit would place the item in bin 7.) If there is a tie, place the item into the bin with the lowest number. Apply this heuristic to the list 8, 7, 1, 9, 2, 5, 7, 3, 6, 4, where the bins have capacity 10.

53. We have described two algorithms for bin packing called worst-fit and best-fit (see page 81 and Exercise 52). The words best and worst have connotations in English. However, the performance of algorithms depends on their merits as algorithms, not on the names we give them.
(a) On the basis of experiments you perform with the best-fit and worst-fit algorithms, which one do you think is the “better” of the two?
(b) Can you construct an example where worst-fit uses fewer bins than best-fit?

54. The best-fit heuristic (see Exercise 52) also has a “decreasing” version, where the list is first sorted in decreasing order. Using bins of capacity 10, apply the best-fit heuristic and its decreasing version to the following list: 6, 9, 5, 6, 3, 2, 1, 9, 2, 7, 2, 5, 4, 3, 7, 6, 2, 3, 7, 1, 6, 4, 2, 5, 3, 7, 2, 5, 2, 3, 6, 2, 7, 1, 3, 5, 4, 2, 5.


56. In the wall-system example in the text, first-fit and worst-fit required equal numbers of bins (see Figure 3.20). Can you find an example where first-fit and worst-fit yield different numbers of bins? Can you find an example where first-fit, worst-fit, and next-fit yield answers with different numbers of bins?

57. A common suggestion for heuristics for the bin-packing problem with bins of capacity W involves finding weights that sum to exactly W. Discuss the pros and cons of a heuristic of this type.

58. A recording company wishes to record all the Beethoven string quartets (16 quartets, each consisting of several consecutive parts called movements) on LPs. It wishes to complete the project on as few records as possible. Recording can be done on two sides as long as the movements are consecutive. Is this an example of a bin-packing problem? (Defend your answer.) If the project were to record the quartets on (standard) tape cassettes or compact discs, would your answer be different?
59. Give examples where it would be realistic to keep bins open as more items "arrive" to be packed, rather than to close a bin permanently based on some criterion.

60. Give examples where it would be unrealistic to keep bins open as more items "arrive" to be packed, rather than to close a bin permanently based on some criterion.

61. A data entry group must handle 30 (independent) tasks that will take the following amounts of time (in minutes) to type: 25, 18, 13, 19, 30, 12, 36, 25, 18, 26, 12, 15, 31, 18, 16, 19, 30, 12, 16, 15, 24, 16, 27, 18, 9, 14. Using these times as a priority list:

(a) Use the list-processing algorithm to find the completion time for scheduling tasks with four secretaries. Also, solve with five secretaries.

(b) Repeat the scheduling using the decreasing-time-list algorithm.

(c) Can you show that any of the schedules that you get are optimal?

If one needs to finish the typing in one hour:

(d) Use the FFD heuristic to find how many typists would be needed.

(e) Repeat for the NFD and WFD heuristics.

(f) Can you show that any of the solutions you get are optimal?

62. Find the minimum number of bins necessary to pack items of size 8, 5, 3, 4, 3, 7, 8, 6, 5, 3, 2, 1, 2, 1, 3, 5, 2, 4, 2, 6, 5, 3, 4, 2, 6, 7, 8, 6, 5, 4, 6, 1, 4, 7, 5, 1, 2, 4 in bins of capacity (a) through (d) using the first-fit and first-fit-decreasing algorithms. Can you determine if any of the packings you get are optimal?

(a) 9
(b) 10
(c) 11
(d) 12

63. Two-dimensional bin packing refers to the problem of packing rectangles of various sizes into a minimum number of m \times n rectangles, with the sides of the packed rectangles parallel to those of the containing rectangle.

(a) Suggest some possible real-world applications of this problem.

(b) Devise a heuristic algorithm for this problem.

(c) Give an argument to show that the problem is at least as hard to solve as the usual bin-packing problem.

(d) If you have 1 \times m rectangles with total area W to be packed into a single rectangle of area p \times q = W, can the packing always be accomplished?

64. In what situations would packing bins of different capacities be the appropriate model for real-world situations? Suggest some possible algorithms for this type of problem.

65. Find an example of weights that, when packed into bins using first-fit, use fewer bins than the number of bins used when the first-fit algorithm is applied with the first weight on the list removed.

66. Formulate "paradoxical" situations for bin packing that are analogous to those we found for scheduling processors.

3.5 Resolving Conflict via Coloring

67. For each of the graphs below:

(a) Color the vertices (if possible) with three different colors.

(b) Color the vertices (if possible) with four different colors.

(c) Find the chromatic number of the graph.

68. For each of the following graphs:

(a)

(b)

(c)

(d)
(a) Color the vertices (if possible) with two different colors.
(b) Color the vertices (if possible) with three different colors.
(c) Find the chromatic number of the graph.

69. The owner of a new pet store wishes to display tropical fish in display tanks. The following table shows the incompatibilities between the species, in the sense that an X indicates that it is unwise to allow those species in the row and column that meet at the X to be in the same tank.

(a) Draw an appropriate graph to represent the information in the table.
(b) What is the minimum number of enclosures needed to avoid housing incompatible animals in the same enclosure?
(c) Is it possible to enclose the animals in such a way that each enclosure contains the same number of animals?
(d) Why might that be desirable? Why might this approach to grouping the animals not be ideal?

71. The nine standing committees of a state legislature are designing a schedule for when the committees can meet. The matrix shown in the following table has an X in a position where the committees corresponding to the row and column have a common member and, hence, should not be scheduled to meet at the same hour. The committees involved are Agriculture (A), Commerce (C), Consumer Affairs (CA), Education (E), Forests (F), Health (H), Justice (J), Labor (L), and Rules (R).

(a) Draw an appropriate graph to represent the information in the table.
(b) What is the minimum number of tanks needed to display all the fish she wishes to sell?
(c) Display the species so that the number of species in each tank is as nearly equal as possible.

70. The managers of a zoo are planning to open a small satellite branch. The animals are to be in enclosures in which compatible animals are displayed together. The accompanying table indicates those pairs of animals that are compatible. (Thus, an X in a particular row and column means that the animals that label this row and column can share an enclosure.)
72. Determine the minimum number of colors, and how often each color is used, in a vertex coloring of the graphs below.

(a)
(b)
(c)
(d)

73. The faculty-student governing council at All State College has nine standing committees (such as Curriculum, Academic Standards, Campus Life) that are designated A, B, C, D, ..., I for convenience. The following table shows which committees have no member in common.

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</table>

(a) Draw an appropriate graph to represent the information in the table.
(b) What is the minimum number of time slots in which all the committee meetings can be scheduled?
(c) How many rooms are needed during each time slot to accommodate the committees that are scheduled to meet in that time slot?

74. When two towns are within 145 miles of each other, the frequency used by a certain type of emergency response system for the towns requires that they be on different frequencies to avoid possible interference with each other. The following table shows the mileage distances between six towns.

<table>
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<th>I</th>
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<tbody>
<tr>
<td>Evansville (E)</td>
<td>290</td>
<td>277</td>
<td>168</td>
<td>303</td>
<td>133</td>
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<tr>
<td>Ft. Wayne (F)</td>
<td>290</td>
<td>132</td>
<td>83</td>
<td>79</td>
<td>201</td>
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<tr>
<td>Gary (G)</td>
<td>277</td>
<td>132</td>
<td>153</td>
<td>58</td>
<td>164</td>
<td></td>
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<tr>
<td>Indianapolis (I)</td>
<td>168</td>
<td>83</td>
<td>153</td>
<td>140</td>
<td>71</td>
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<tr>
<td>South Bend (S)</td>
<td>303</td>
<td>79</td>
<td>50</td>
<td>140</td>
<td>196</td>
<td></td>
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<tr>
<td>Terre Haute (T)</td>
<td>113</td>
<td>201</td>
<td>164</td>
<td>71</td>
<td>196</td>
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</table>

(a) What would be the minimum number of frequencies that are needed for each town to have its emergency broadcasts not conflict with those of any other town using this system?
(b) How many different towns would be assigned to each frequency used?

75. Show that the vertices of any tree can be colored with two colors.

76. Can you find a family of graphs $H_n$ ($n \geq 1$) that require $n$ colors to color their vertices?

77. The edge-coloring number of a graph $G$ is the minimum number of colors needed to color the edges of $G$ so that edges that share a common vertex get different colors. Determine the edge-coloring number for each of the graphs in Exercise 67. Can you make a conjecture about the value of the minimum number of colors needed to color the edges of any graph?

78. Can you think of any applications that require determining the minimum number of colors needed to color the edges of a graph?

79. When a graph has been drawn on a piece of paper so that edges meet only at vertices, the graph divides the paper up into regions called faces. The faces include one called the "infinite" face, which surrounds the whole graph. The face-coloring number of a graph $G$ (which can be drawn in this special way) is the minimum number of colors needed to color the faces of $G$ so that two faces that share an edge receive different colors. (Note that if two faces meet only at a vertex, they can be colored the same color.)

(a) Determine the minimum number of colors needed to color the faces of the following graphs. In each case, remember to color the infinite face, which is labeled $I$ (for "infinite").
PART I Management Science

For each of the graphs in Exercise 68 where the graph shown has edges that meet only at vertices, verify that the Four Color Theorem holds by showing that the regions (faces) of the graph can be colored with four or fewer colors so that regions that share an edge get different colors. (Remember to assign a color to the unbounded, so-called infinite region.)

81. A company sells herbs, each of which requires a certain level of proper watering. The following graph is constructed by having one vertex for each type of herb. The vertices representing two herbs are joined by an edge if they must have different levels of watering. What is the minimum number of terrariums that the herbs can be displayed in so that herbs in the same terrarium can be watered at the same level?

(b) Can you think of an application of the problem of coloring the faces of a graph with a minimum number of colors?

82. The company in Exercise 81 is disappointed by the minimum number of terrariums needed to display the herbs with the proper watering requirements. One company employee suggests that if the information about watering requirements is altered for a single pair of herbs (e.g., a single edge is erased from the diagram), then the number of terrariums needed will be reduced by 1. Is this true?

83. Each vertex in the graph below represents a child who attends a day care center. An edge between two children indicates these children tend to cause problems when they are in the same play group. What is the minimum number of play groups that will ensure that no conflicts arise? Can conflict-free play groups with the same number of children in each group be formed?

**APPLET EXERCISES**

To do these exercises, go to www.whfreeman.com/fapp8e.

**Graph Coloring**

Solving a scheduling problem such as the one below can be accomplished by constructing a related graph and then coloring it in a way that adjacent vertices have different colors. Explore the problem of graph coloring in the Graph Coloring applet.

**Scheduling**

A mathematics department has seven faculty committees—A, B, C, D, E, F and G. Because there is overlap in the composition of the committees, the chairman of the department is attempting to work out a schedule that will avoid conflicts among the committees. The following chart indicates the overlapping committee structure:

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Help the chairman arrange a schedule without conflicts in the Scheduling applet.